Word: convex Word Look it up

Convex

An object is **convex** if for any pair of points within the object, any point on the line that joins them is also within the object. For example, a solid cube is convex, but anything that is hollow or has a dent in it is not convex.

Convex set

In mathematics, convexity can be defined for subsets of any real or complex vector space. Such a subset C is said to be **convex** if, for all x and y in C and all t in the interval [0,1], the point $tx + (1-t)y$ is in C. In words, every point on the straight line segment connecting x and y is in C

The convex subsets of **R** (the set of real numbers) are simply the intervals of **R**. Some examples of convex subsets of Euclidean 3-space are the Archimedean solids and the Platonic solids. The Kepler solids are examples of non-convex sets.

The intersection of any collection of convex sets is itself convex, so the convex subsets of a (real or complex) vector space form a complete lattice. This also means that any subset A of the vector space is contained within a smallest convex set (called the convex hull of A), namely the intersection of all convex sets containing A.

If one restricts to closed convex sets, they can actually be characterised as the intersections of closed half-spaces, lying to one side of a hyperplane. From what has just been said, it is clear that such intersections are convex, and they will also be closed sets. For the converse, one need the *supporting hyperplane theorem* in the form that for a given closed convex set C and point P outside it, there is a closed half-space H that contains C and not P. (That theorem is a case of the Hahn-Banach theorem of functional analysis, but less deep.)

Convex function

A real-valued function f defined on an interval (or on any convex subset of some vector space) is called **convex** if for any two points x and y in its domain and any t in [0,1], we have :f(tx+(1-t)y)\\leq t f(x)+(1-t)f(y).

A function is also said to be **strictly convex** if :f(tx+(1-t)y) < t f(x)+(1-t)f(y).

One may compare this definition of convexity and that for sets, and note that a function is convex if, and only if, the region of the plane lying above the graph of said function is a convex set.

Properties of convex functions

A convex function defined on some open set is continuous on the whole interval and differentiable at all but at most countably many points. A twice differentiable function of one variable is convex on an interval if and only if its second derivative is non-negative there and strictly convex if and only if its second derivative is positive; this gives a practical test for convexity.

Any local minimum of a convex function is also a global minimum. A *strictly* convex function will have at most one global minimum.

A convex function respects the Jensen's inequality.

Examples of convex functions

- The second derivative of x^2 is 2; it follows that x^2 is a convex function of x.
- The absolute value function $|x|$ is convex, even though it does not have a derivative at $x = 0$.
- The function $f(x) = x$ is convex but not strictly convex.
- The function x^3 has second derivative $6x$; thus it is convex for $x \geqslant 0$ and concave for $x \leqslant 0$.